TRINITY COLLEGE

Mathematics Methods Unit 3/4
Test 5 2022

Section 1 Calculator Free

CRVs and Normal Distribution

STUDENT'S NAME _____

DATE: Tuesday 9th August **TIME:** 15 minutes **MARKS**: 14

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Let X be a normally distributed random variable with a mean of 6 and a variance of 4. Let Z be a random variable with the standard normal distribution.

(a) Determine $P(X < 8)$. [2]

(b) Determine b such that $P(X < 7) = P(Z < b)$. [1]

2. (5 marks)

The probability density function for a continuous random variable X is given by

$$f(x) = \begin{cases} ax(4-x) & 0 \leq x \leq 4 \\ 0 & elsewhere \end{cases}$$

(a) Show that the value of a is $\frac{3}{32}$.

[3]

(b) Determine $P(X < 3)$ [2]

3. (6 marks)

The continuous random variable X takes values in the interval 1 to 5 and has cumulative distribution function $F(x)$ where

$$F(x) = P(X \leq x) = \begin{cases} 0 & x < 1 \\ \frac{x-1}{4} & 1 \leq x \leq 5 \\ 1 & x > 5. \end{cases}$$

(a) Determine

(i) $P(X \leq 3.5)$ [1]

(ii) the value of k, if $P(X > k) = 0.85$ [2]

(b) Determine $f(x)$, the probability density function of X, and sketch the graph of $y = f(x)$. [3]

TRINITY COLLEGE

Mathematics Methods Unit 3/4
Test 4 2022

Section 2 Calculator Assumed

CRVs and Normal Distribution

STUDENT'S NAME _____

DATE: Tuesday 9th August **TIME:** 25 minutes **MARKS**: 27

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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4. (4 marks)

The continuous random variable, X, is normally distributed with $P(X < 28) = 0.35$

(a) How many standard deviations from the mean is a score of 28? [2]

(b) If the standard deviation of X is 5.74 find the mean of the distribution, giving your answer correct to 2 decimal places. [2]

5. (4 marks)

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{x}{12} & 1 \leq x \leq 5 \\ 0 & elsewhere \end{cases}$$

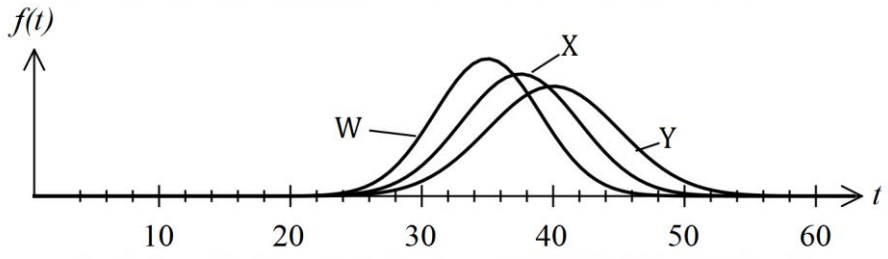
(a) Explain why f is a probability density function. [1]

(b) Determine $P(X < 3)$. [1]

(c) If $P(X \geq a) = \frac{5}{8}$, find the value of a. [2]

6. (8 marks)

The graphs of the probability density functions of three normally distributed random variables W, X and Y are shown below.



(a) State, with justification, which of the three random variables has

(i) the largest standard deviation? [1]

(ii) the largest mean? [1]

(b) Empty bottles are filled with A mL of water, where A is a normally distributed random variable with mean of 510 mL and standard deviation of 7.5 mL.

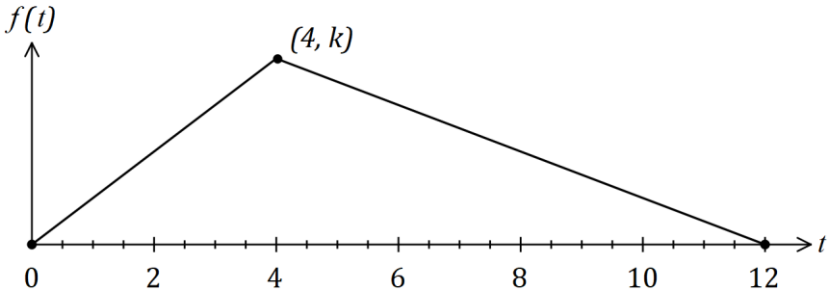
(i) Determine the probability that a bottle is filled with more than 520 mL. [1]

(ii) Determine the probability that a bottle is filled with less than 515 mL, given that it is filled with more than 510 mL. [2]

(iii) The mean of A is to be decreased by k mL so that just 2.5% of bottles are filled with 520 mL or more. Determine the value of k. [3]

7. (11 marks)

The time T to process orders at a warehouse is a random variable which can take any value in the interval 0 to 12 minutes. The graph of the triangular probability density function of T is shown below.



(a) Determine the value of k. [1]

(b) Determine the probability that the time to process an order takes less than 3 minutes. [3]

(c) Determine the mean time to process an order in minutes and seconds. [4]

The variance of T is 6 minutes 13 seconds.

(d) Two new procedures will affect the processing time of an order. The first will decrease the time by 15% and the second will then add one-and-a-half minutes. Determine the new mean and standard deviation of the time to process an order. [3]